A compositional theory of digital circuits

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Digital circuits!





















(but these are a little different to the ones Dan and Jamie are using)















This lets us build combinational circuits.



These circuits are boring!



E- 5-

E- S- b-

E- 5- 6- ->-

##
These circuits are boring! Circuits need state...

...and feedback!



These circuits are boring! Circuits need state...

...and feedback!



Now what?

Now what?

None of this means anything!

Open the gates

AND = VANT





10

₽-< = ₽ **→** _ _

Time to join up













We must not delay



This gives us some nice structure

From axioms we derive theorems

From axioms we derive theorems



From axioms we derive theorems





We want to process inputs to obtain outputs

We want to process inputs to obtain outputs

SE->-GS

This axiom is the last one

This axiom is the last one



This axiom is the last one

















Arbitrary gate and value sets...

Arbitrary gate and value sets... Handling non-delay-guarded feedback...

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Defining the denotational semantics of circuits as stream functions...
We're only scratching the surface!

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Defining the denotational semantics of circuits as stream functions... Interpreting circuits as hypergraphs for reasoning computationally...

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(someone could write this up as a PhD thesis)